Understanding Sampling Distributions: The Role of Interactive Dynamic Technology

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- What image comes to mind when you think about a distribution of a set of data?

- What image comes to mind when you think about the distribution of a set of data?
- Random?

A concept image

- can be described as the total cognitive structure including the mental pictures and processes associated with a concept built up in students' minds through different experiences associated with the ideas (Tall & Vinner 1981).
- is necessary to fluently and effectively reason with and apply ideas; without a coherent mental structure, students are left to construct an understanding based on ill formed and often misguided connections and images (Oehrtman 2008).

Concept Images

- Premise: Interactive dynamic technology has the potential to help students build robust concept images in mathematics.

Students can evoke “movie clips” of dynamic experiences that replay in their minds when they encounter words, graphs, equations,… related to a concept such as variable, expression, equation, solution

From “I remember seeing this...” To “I remember seeing this *happening*...”)

Action Consequence Principle

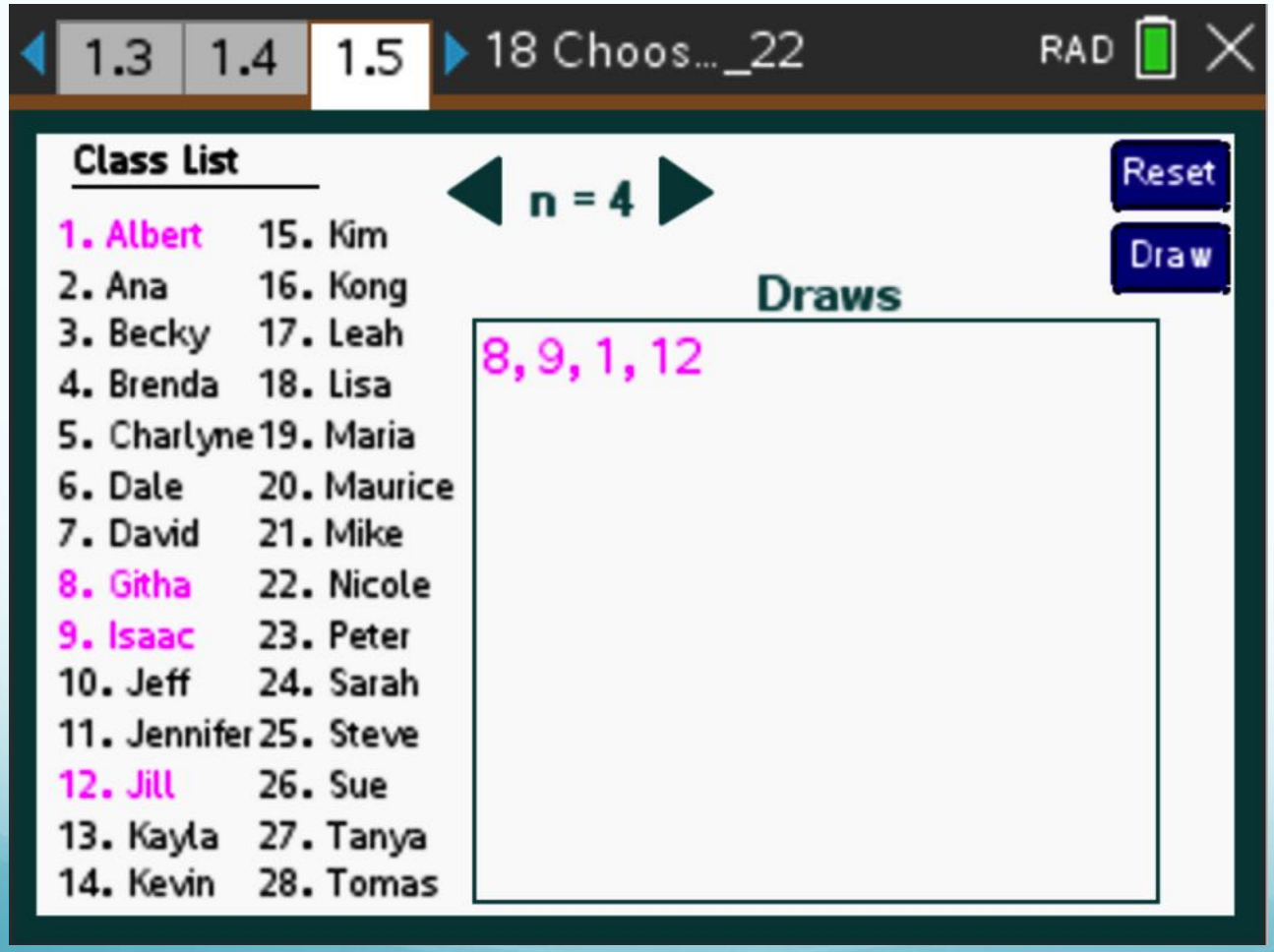
- The learner deliberately takes a statistical (mathematical) action, observes the consequences, and reflects on the statistical implications of the consequences.

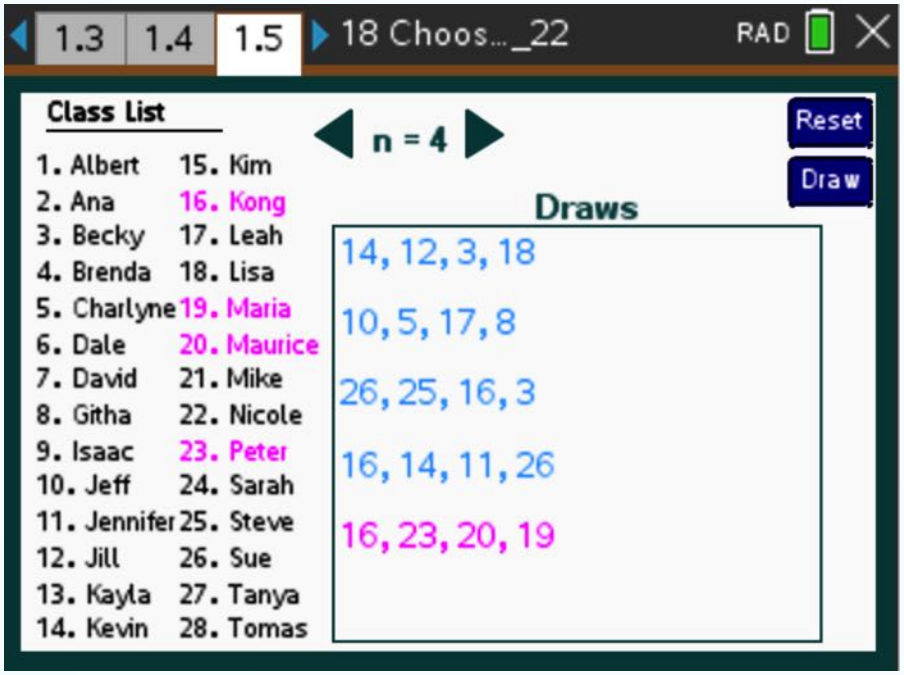
- Interactive dynamic technology provides an opportunity for students to visualize the action and the consequences, which can enable them to create a dynamic mental image of the concept.

Students should have experiences that

- Create a mental construct or image of the concept as basis for thinking
- Add to this image in robust ways
- Build understanding and confront misconceptions
- Include attention to metacognition and flexibility in thinking

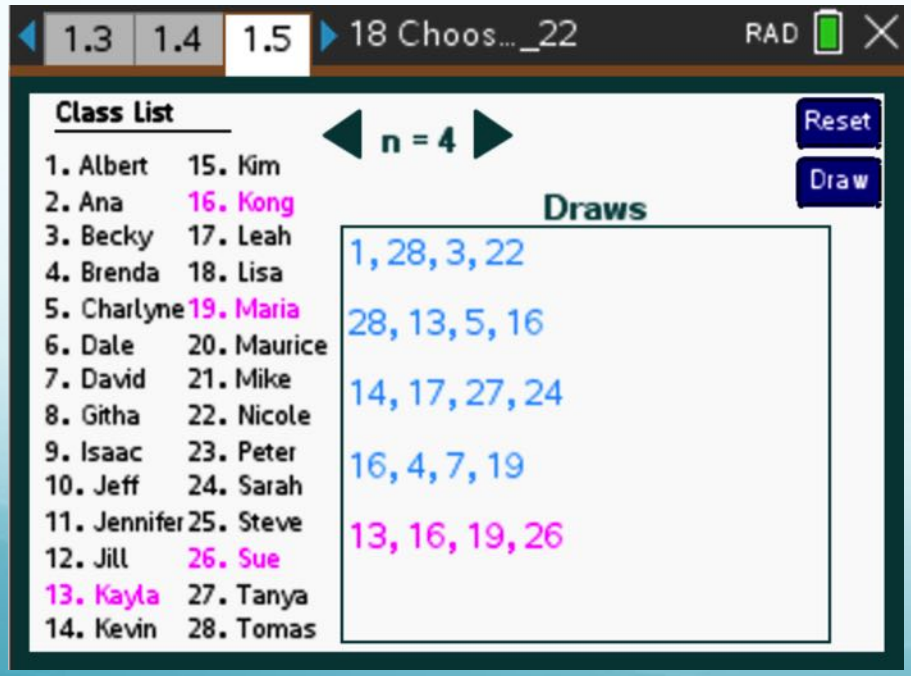
What is a random sample?

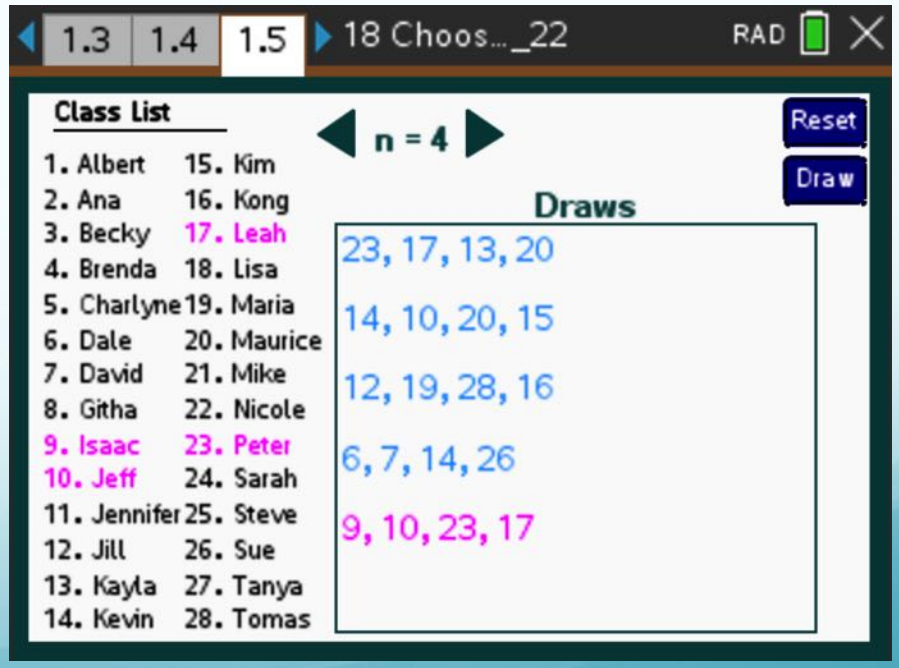




What do you notice?

Is random "fair"?

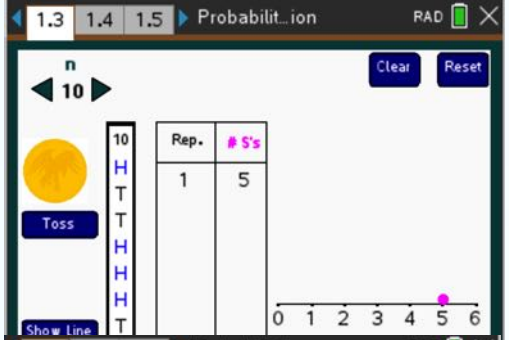


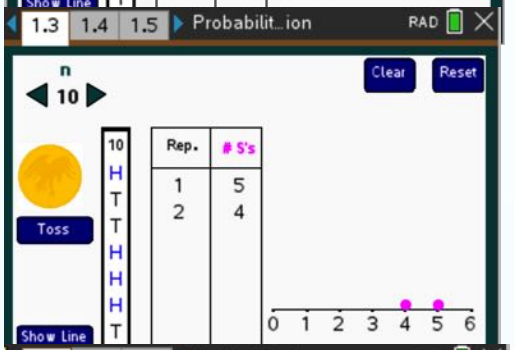


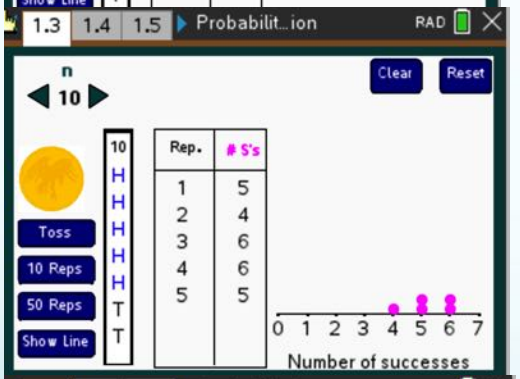
Building a distribution

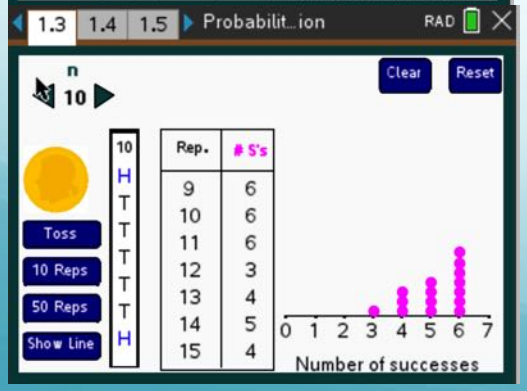
- What are the chances of passing a ten question true false test by guessing?

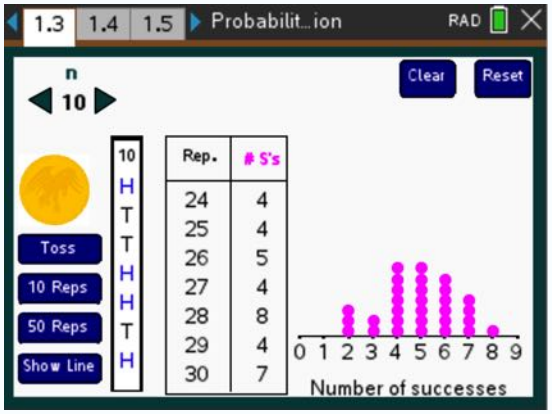
Building a sampling distribution

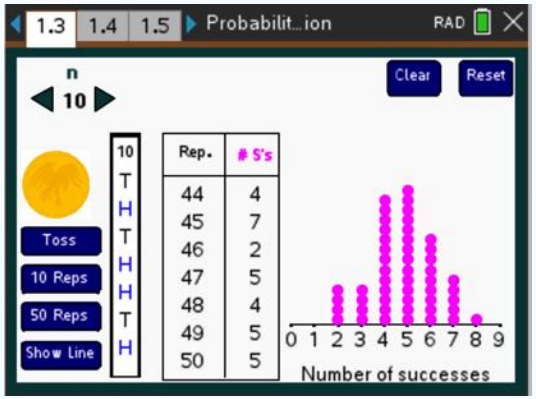


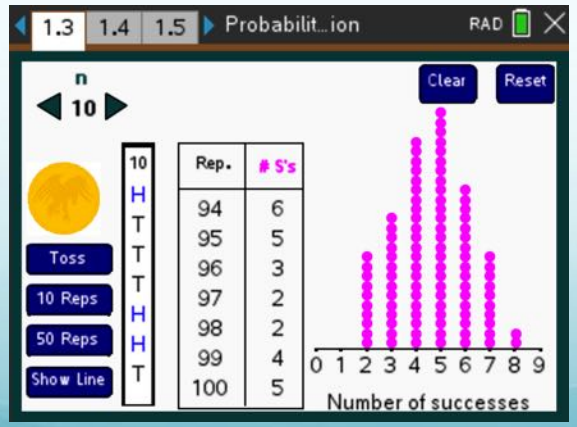


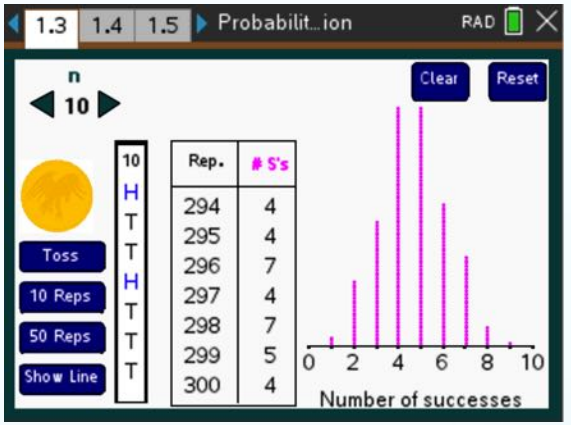


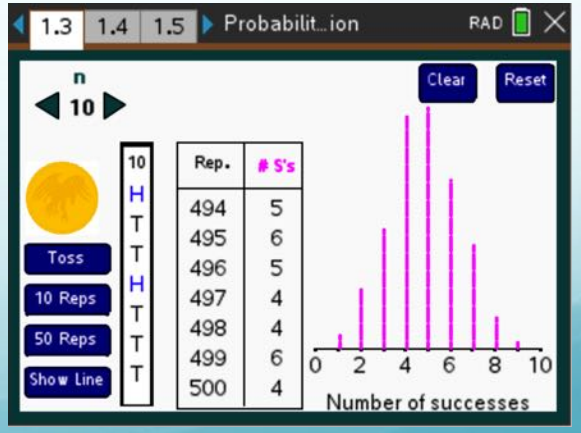












Important concepts in understanding sampling distributions

- Distribution- Collection of data showing frequency of values and typically represented graphically
  - Mean as fair share and as balance point
  - Variability in distributions
    - Deviations from the mean (mean absolute deviation/ standard deviation)
- Sampling
  - Random
    - Population and representativeness
  - Variability in sampling
    - From sample to sample
    - Within samples
- Sampling distributions

Instructional activities supporting the development of concept images

- The underlying structure that is the target for student learning should be reflected in the actions they do.
- Students' actions should be repeated and organized with provisions for feedback and ways to respond to this feedback.
- Students should repeat these actions in structurally similar problems in a variety of contexts to develop a robust abstraction of the concept. (Oehrtman, 2008)

Misconceptions related to distributions

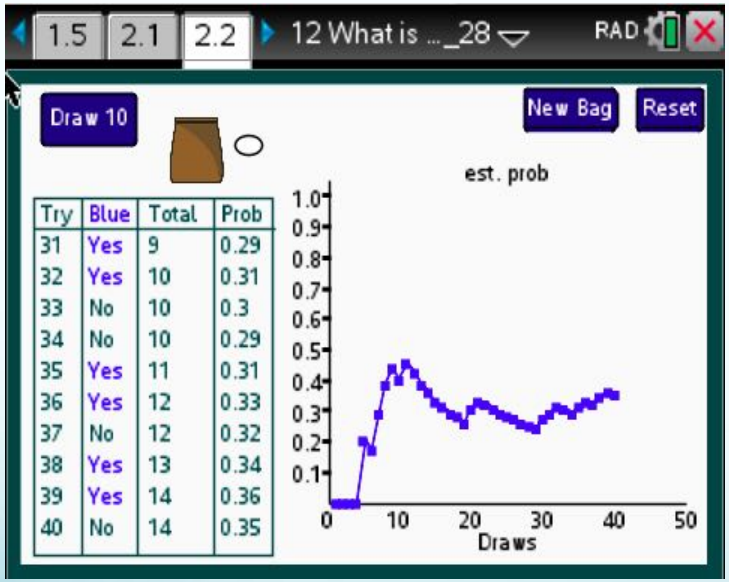
- Confuse distributions of a population, a sample from the population, and a sampling distribution of a sample statistic (Wild, 2006)
- Believe sampling distributions for small and large sample sizes have same variability
- Believe a random sample is a model of the population; don't consider the variability across all possible samples, and how their sample might fit into that range of possibilities (Chance, delMas, & Garfield 2004).
- Assume two samples from the same population will be similar (Tversky & Kahneman, 1971)
- Assume the sampling distribution will look like that of the population (for $n > 1$)
- Not distinguishing between a frequency distribution and a relative frequency distribution

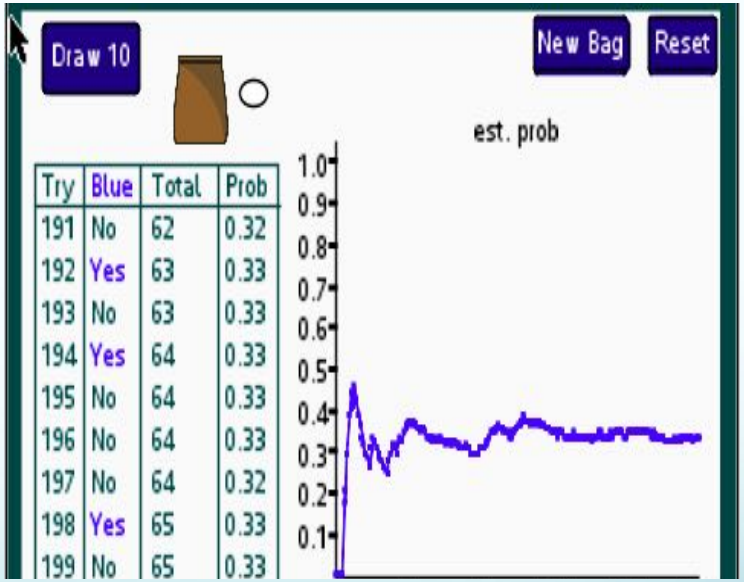
The Project: The Role of Interactive Dynamic Technology in Teaching and Learning Introductory Statistics

- Course for elementary preservice students in statistics and probability
- Backgrounds of students varied from no statistics course to AP statistics to an intro university statistics course
- Students had identified an emphasis on mathematics in their program
- Used TI Nspire software https://education.ti.com/en/building-concepts/ activities/statistics) and Statkey (www.lock5stat.com/StatKey/).

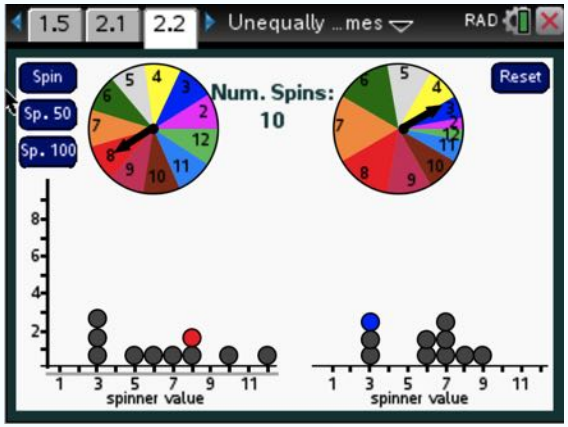
Variability in small and large samples

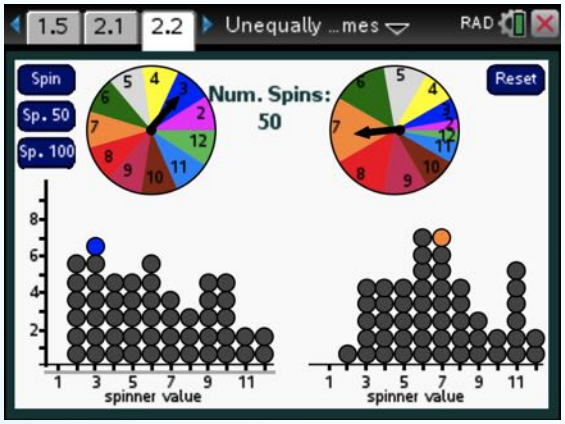
What proportion of the chips in the bag are blue?



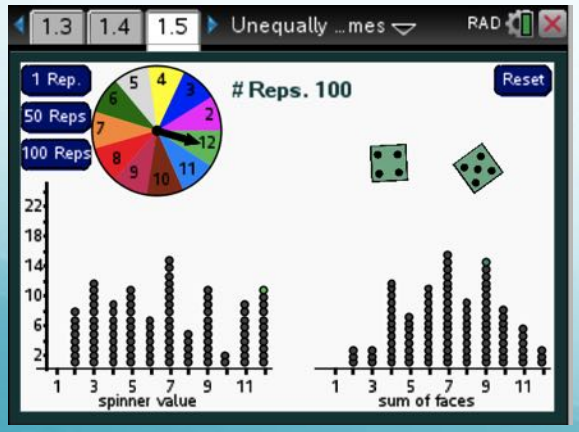


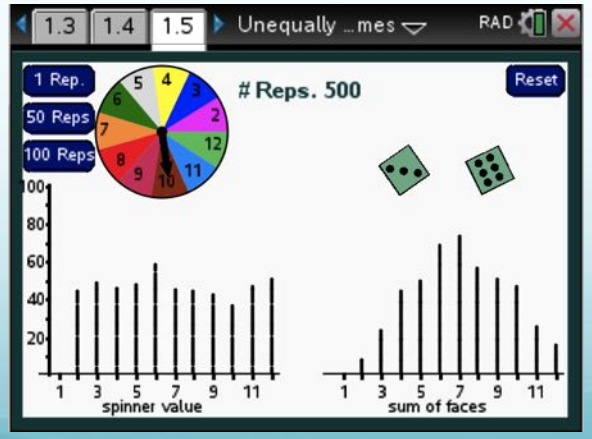
Revisiting sample variation

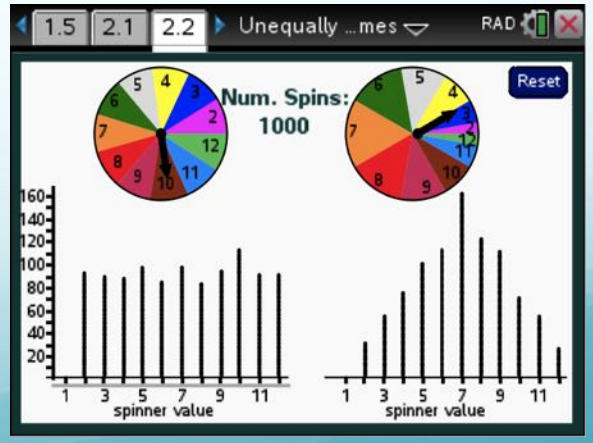




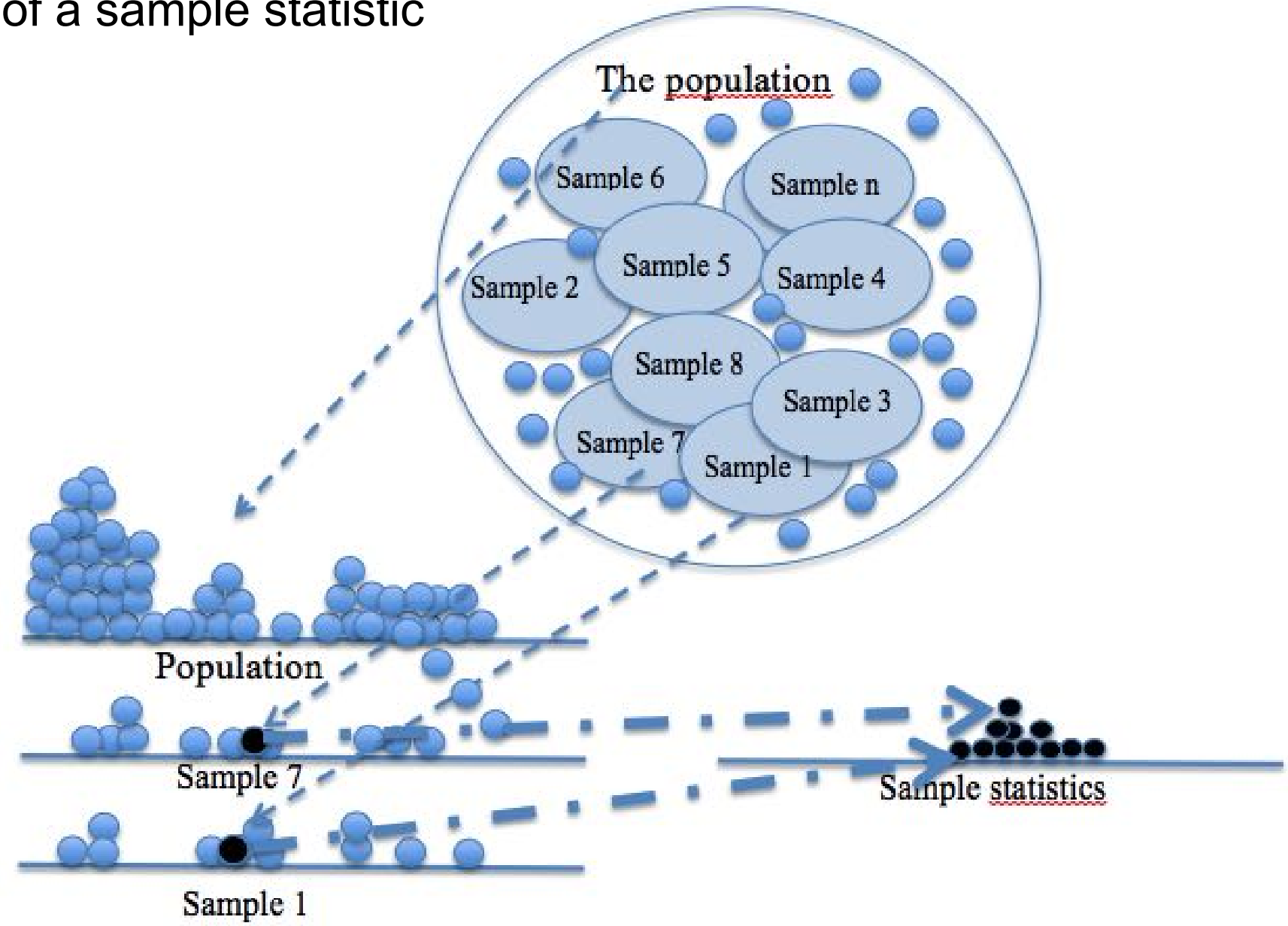
The distribution of random samples for a small *n* vary a lot, but as *n* increases, the distributions "settle down"

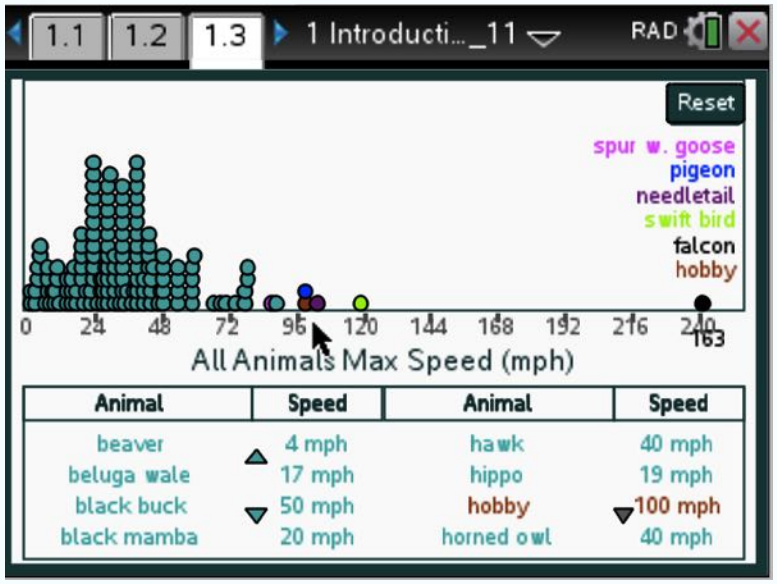






Distinguishing distributions of a population, a sample and of a sample statistic

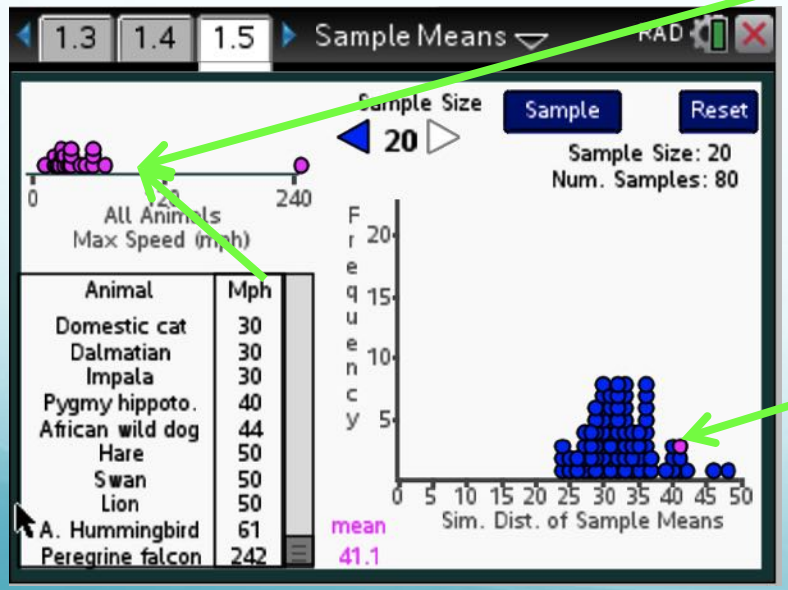




Population: speeds of assorted animal types

Variability in sampling:

Distribution of maximum recorded speeds of 20 randomly selected animal types



Sampling distribution of means from each sample

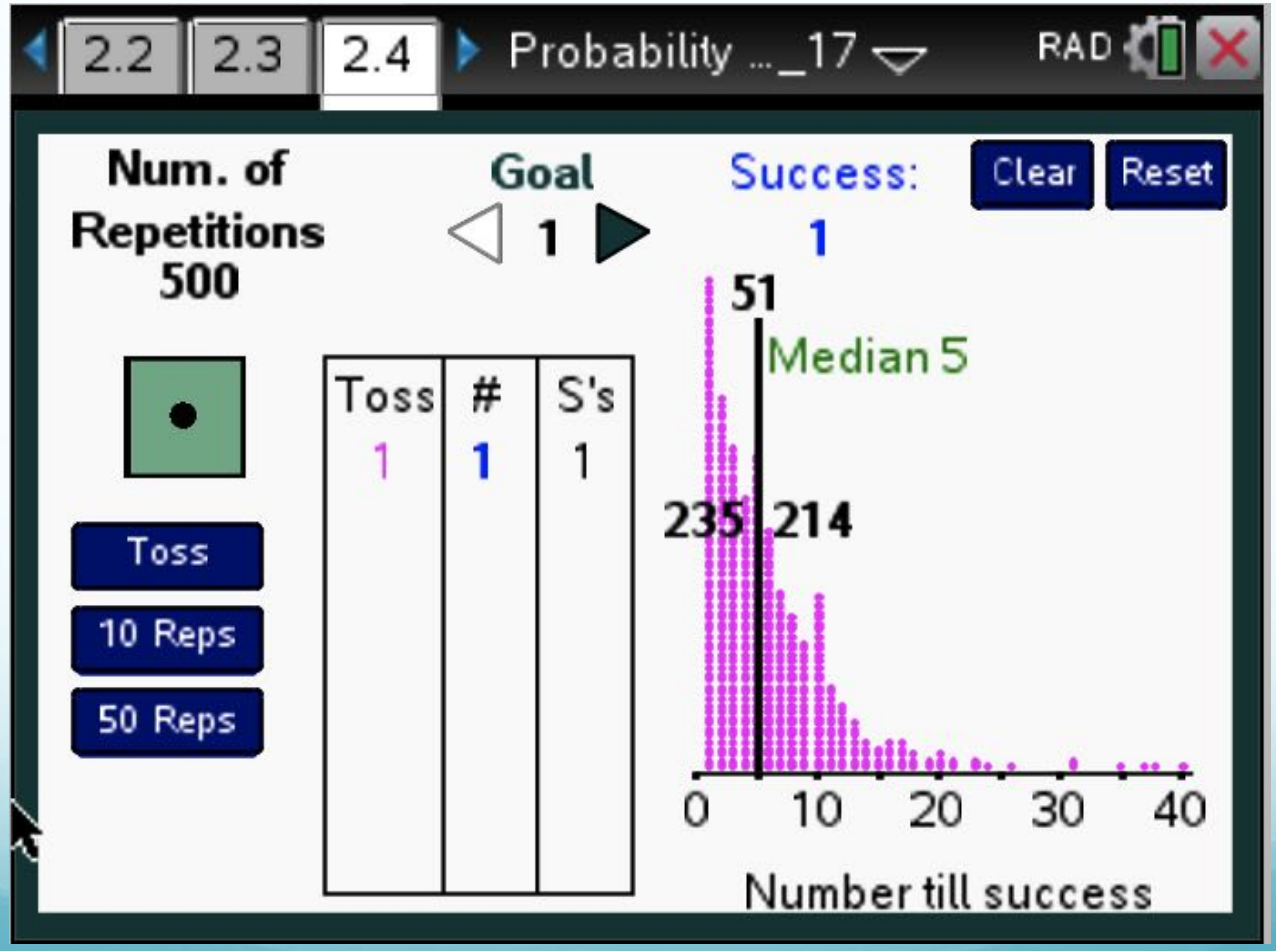
Revisiting distributions

A cereal company puts a toy in every sixth box of cereal.

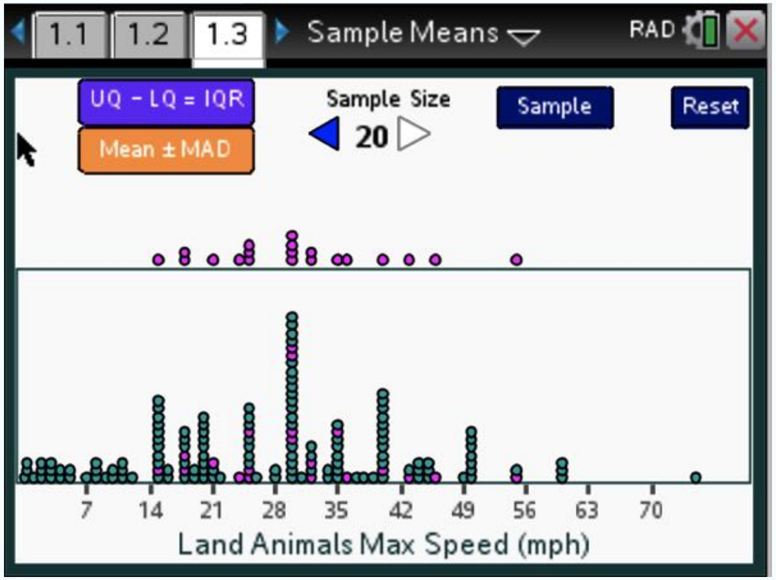
a) Do you think if you buy six boxes, you will get the prize? Why or why not?

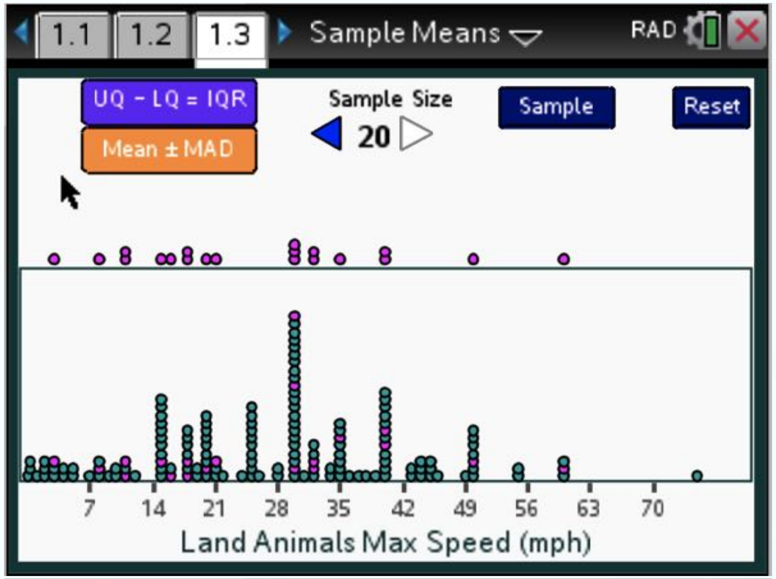
b) Estimate how many boxes of cereal you would have to buy before you got the prize.

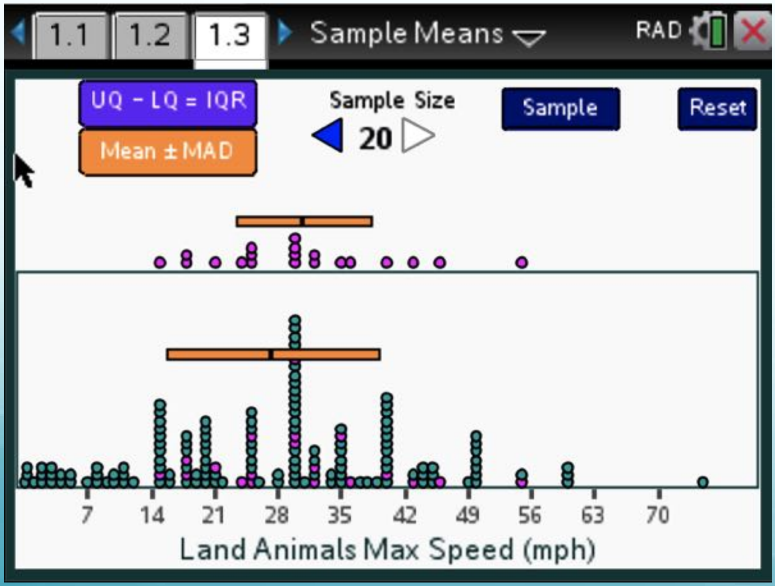
A cereal company puts a toy in every sixth box of cereal. Do you think if you buy six boxes, you will get the prize? Why or why not? Estimate how many boxes of cereal you would have to buy before you got the prize.

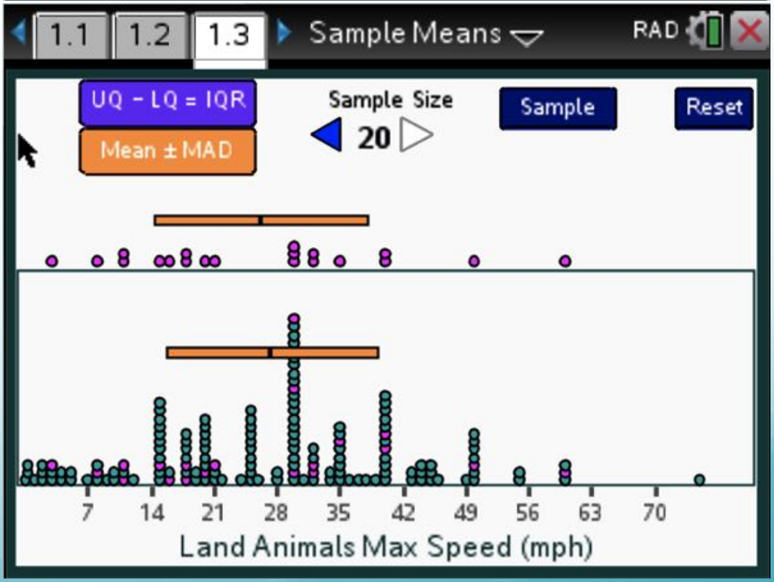


Revisiting populations/samples

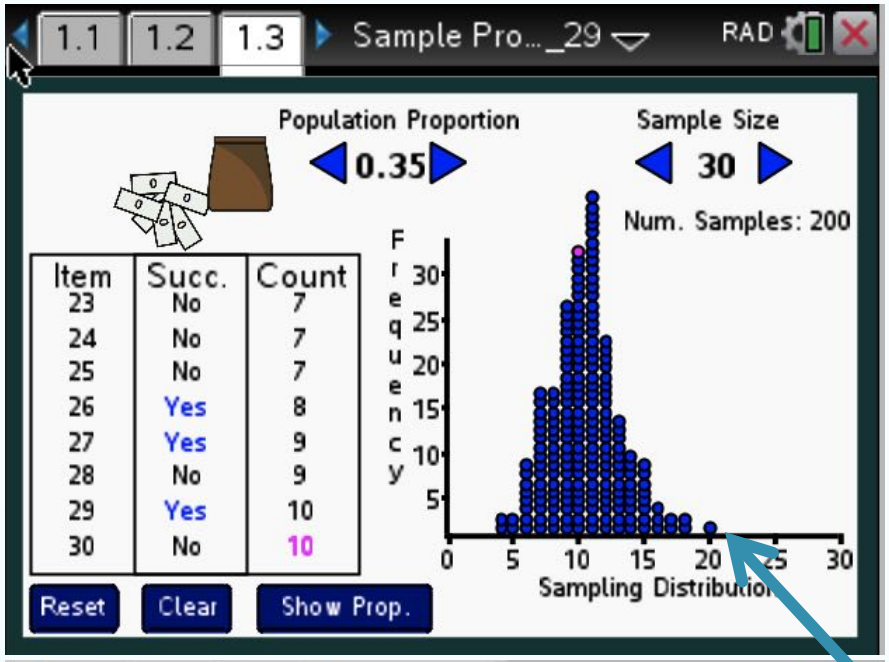


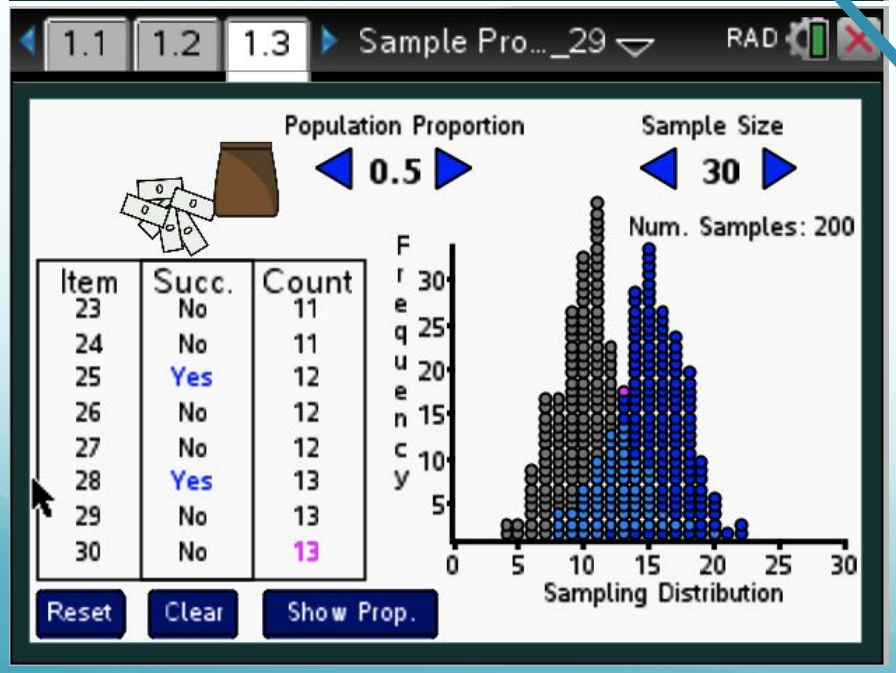






Building Concepts, Probability and Statistics: Sample Means, 2015

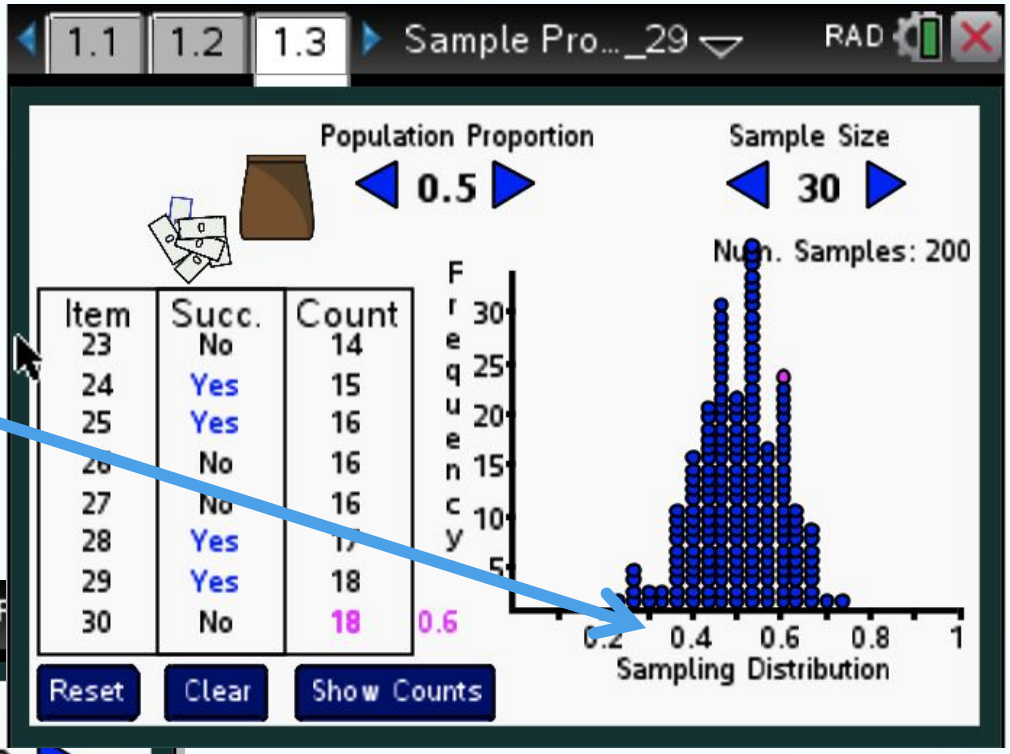


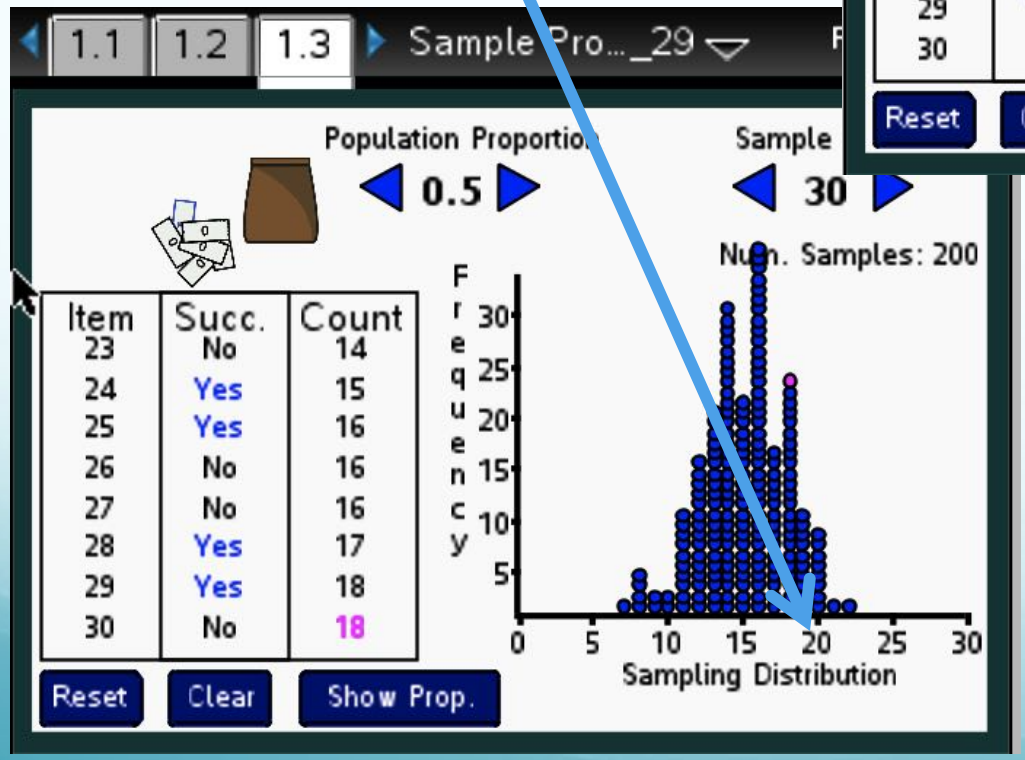


Chance variability: simulated sampling distributions for given population proportions and sample sizes

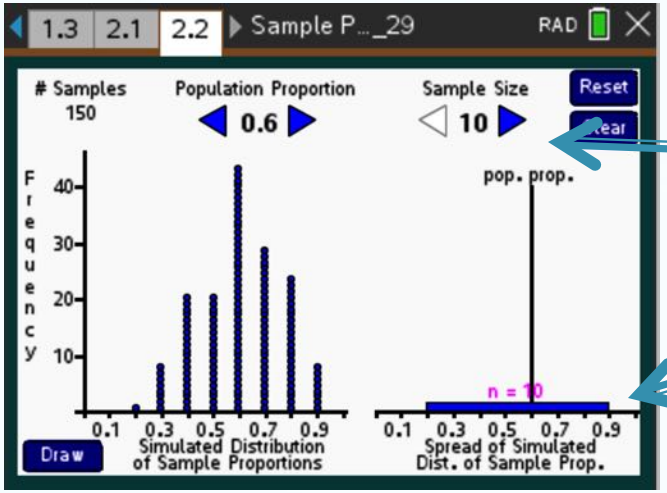
How likely is an outcome of 20 successes in a sample of size 30 for a known population proportion of 0.35?

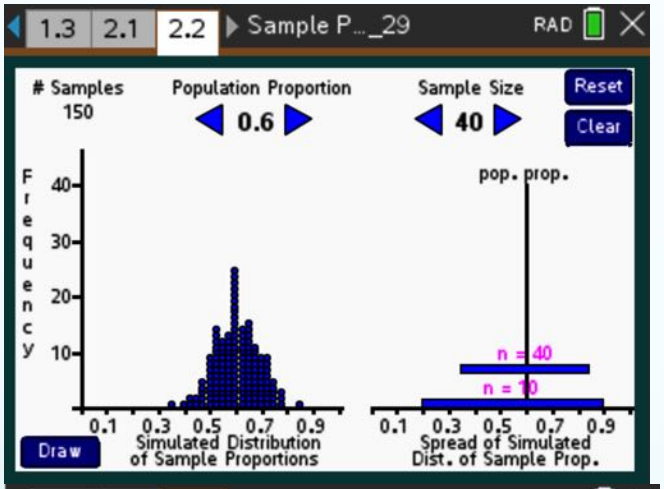
Counts vs proportions

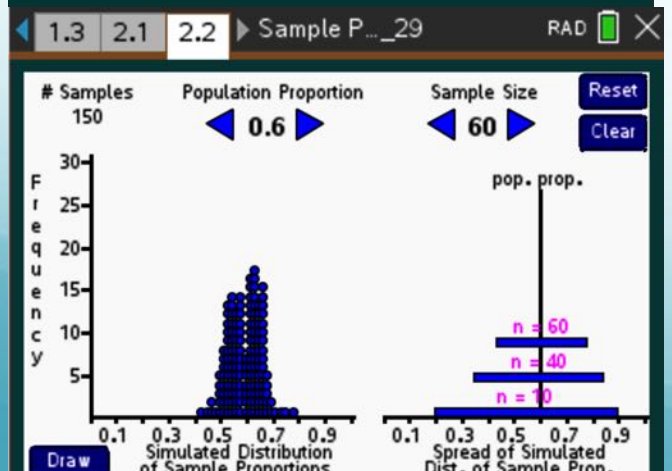


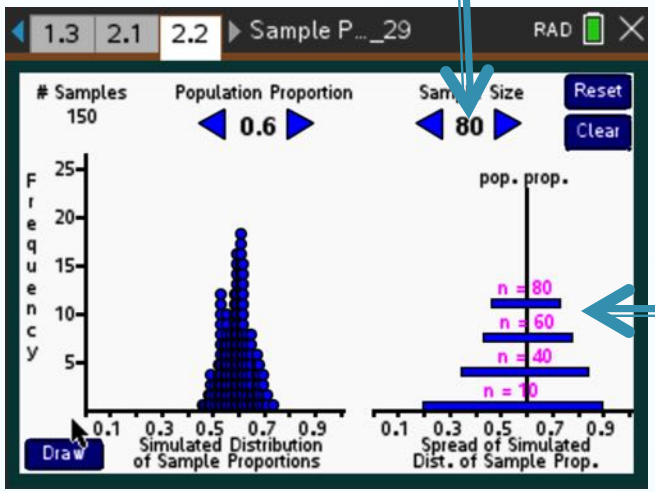


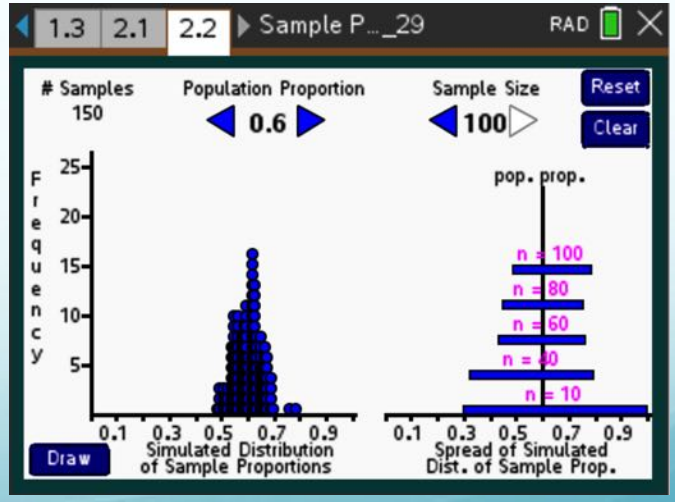
Sample size and variability





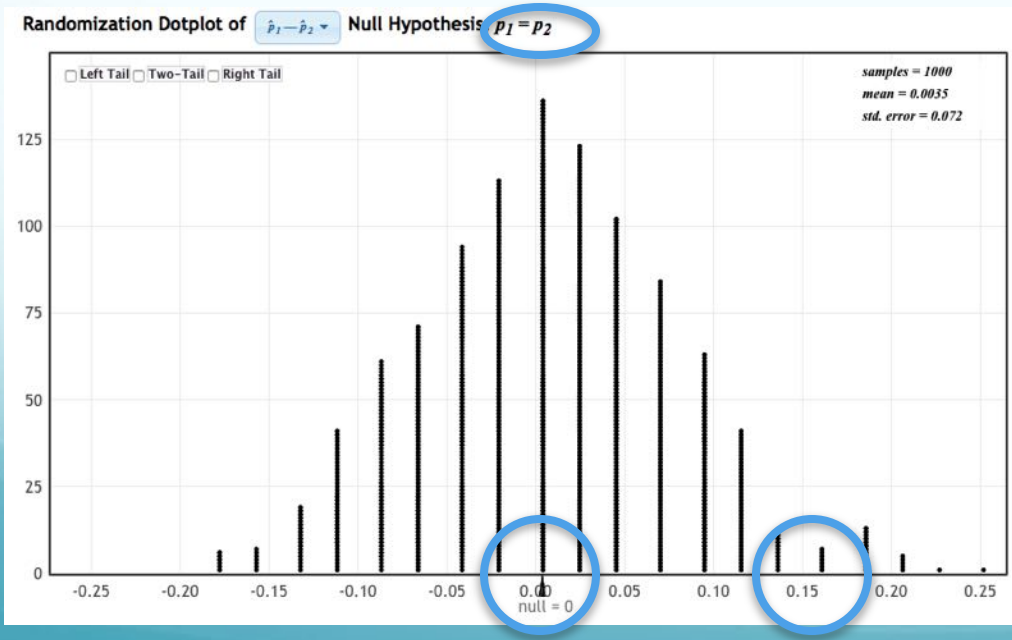






Building Concepts: Statistics and Probability, Sample Proportions, 2016

- Carpal tunnel syndrome can be treated with surgery or less invasive wrist splints. A study of **176** patients found that among the **half** that had surgery, **80%** showed improvement after three months. Only **54%** of those who use the wrist splints improved. Is there evidence of a real difference between the two proportions or could the difference have occurred by chance? Why or why not?(Song, 2002)



Original Sample

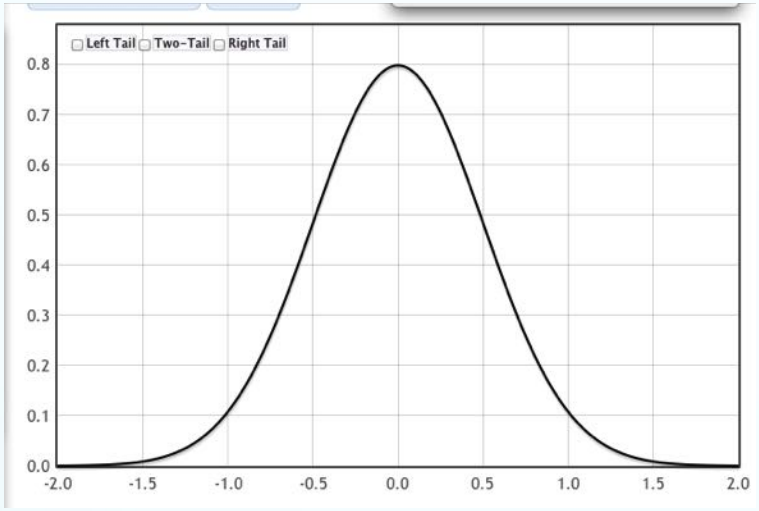
Group	Count	Sample Size	Proportion
Group 1	70	88	0.795
Group 2	48	88	0.545
Group 1–Group 2	22	n/a	0.25

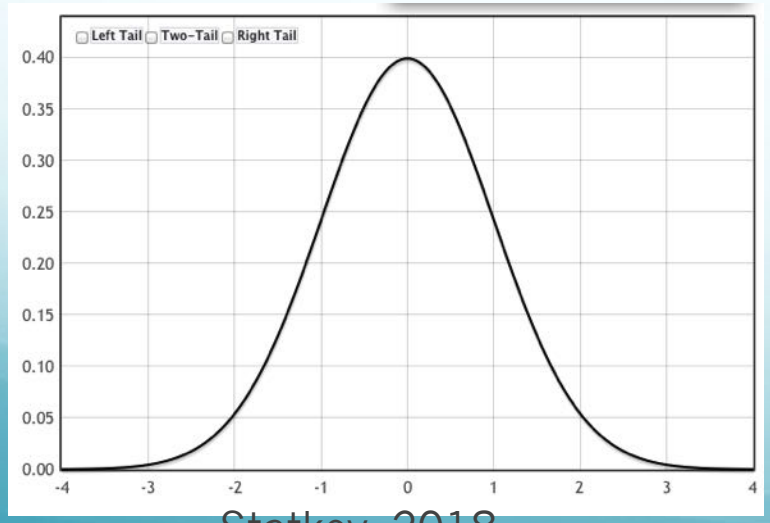
Randomization Sample

Group	Count	Sample Size	Proportion
Group 1	61	88	0.693
Group 2	57	88	0.648
Group 1–Group 2	4	n/a	0.045

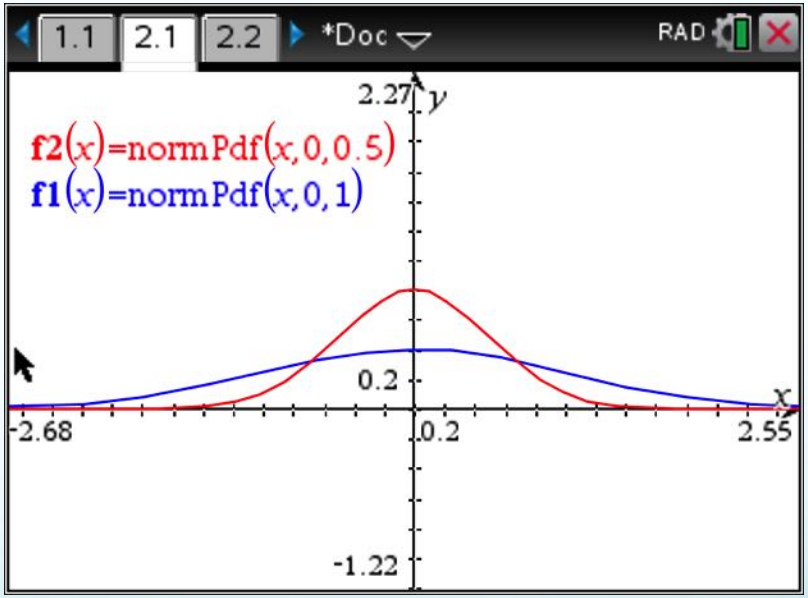
Statkey, 2018

And the difference is…?





Statkey, 2018



TI Nspire

Analysis: SOLO Taxonomy (Structure of Observed Learning Outcomes, Biggs & Collis, 1982)

Level of SOLO taxonomy	**Description of application to concept image**
Prestructural (P)	Does not refer to key elements of the concept.
Unistructural (U)	Focuses on one key element of the concept.
Multistructural (M)	Focuses on more than one key element of the concept.
Relational (R)	Develops relational links between various key elements of the concept.

SOLO taxonomy and concept images adapted from Reading & Reid (2006)

Features associated with concept images related to sampling distributions

SOLO taxonomy level	Concept images for relating sampling distributions to populations and samples
Prestructural (P)	Does not distinguish between a population, a sample from that population and the distribution of a sample statistic
Unistructural (U)	Identifies/interprets simulated sampling distribution of a sample statistic
Multistructural (M)	Recognizes that sampling distributions of sample means/proportions will be a normal distribution
Relational (R)	Links sample size to variability, visually compares distributions of sample proportions/means with respect to sample size; uses a simulated sampling distribution to consider whether an observed outcome is likely

Question on second test

Half of all newborns are girls, and half are boys. Hospital A records an average of 50 births per day. Hospital B records an average of 10 births a day. On a particular day, which hospital is more likely to record 80% or more of female births?

a. Hospital A (with 50 births a day)

b. Hospital B (with 10 births a day)

c. The two hospitals are equally likely to record such an event.

- Please explain your reasoning. (Reaburn, 2008)

Half of all newborns are girls, and half are boys. Hospital A records an average of 50 births per day. Hospital B records an average of 10 births a day. On a particular day, which hospital is more likely to record 80% or more of female births?

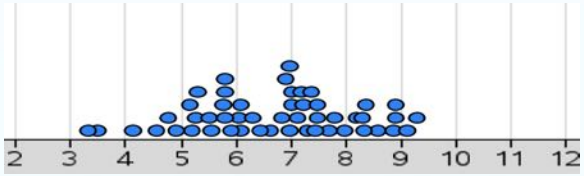
a. Hospital A (with 50 births a day)
b. Hospital B (with 10 births a day)
c. The two hospitals are equally likely to record such an event.

SOLO taxonomy level	Concept images for relating sampling distributions to populations and samples
Prestructural (P)	38% offered explanations in the P or U levels such as: ““the two hospitals are equally likely because 80% is a proportion.”
Unistructural (U)	
Relational (R)	70% (as opposed to Raeburn 46%) linked sample size to variability in samples: “Hospital B (with 10 births a day) – Because the sample size is smaller they’re more likely to record a higher percentage of girl births. A small sample size allows for more variation in responses and are more likely to have extreme results whereas a larger sample size is more likely to be true to the actual average of female and male births.”

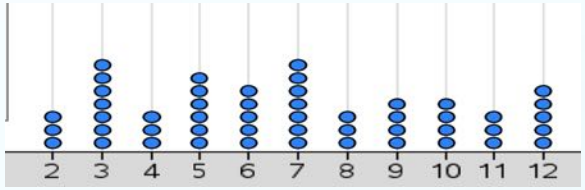
Final exam and distributions: population, sample and sample means

Population

Sample?

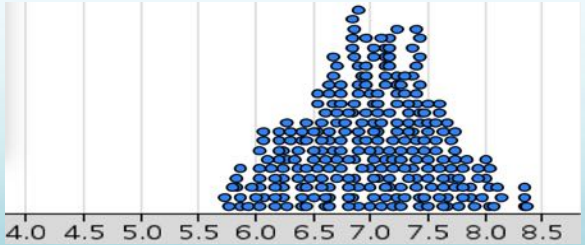


a



b

Sample means?



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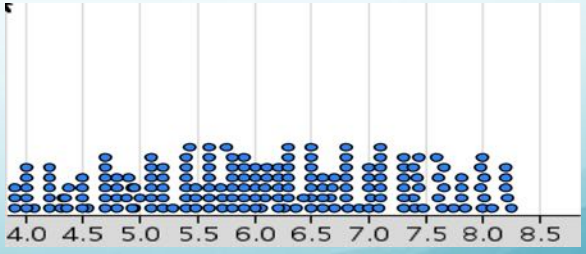

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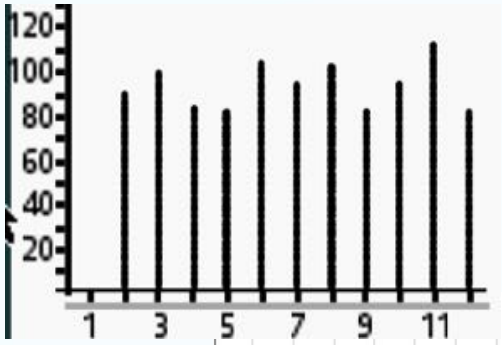
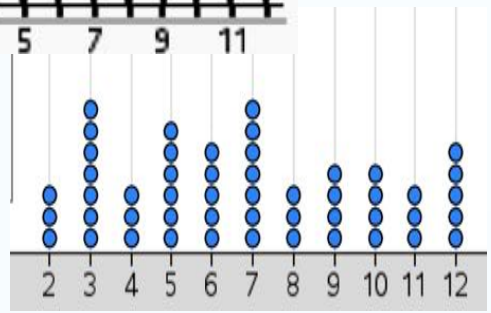
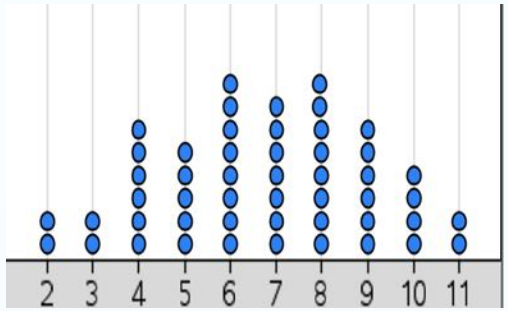




a) The two plots are from random samples of size 50. Which of the two distributions was more likely to be a *random sample* from the population in figure 8?



SOLO taxonomy level	Concept images for relating sampling distributions to populations and samples
Prestructural (P)	In a), one student was unable to identify a sample: "Neither figure is more likely to be a random sample from the population because if they are random they each have an equally likely chance of occurring."
Multistructural (M)	For a) 92% connected a sampling distribution to a population: "This is because when you take random samples from the population above and plot them below it is likely that the distribution will look similar. This figure and the sample are both uniform and if the population was sampled it would likely end up being uniform because points come from anywhere in the graph so the graph will keep a similar distribution."

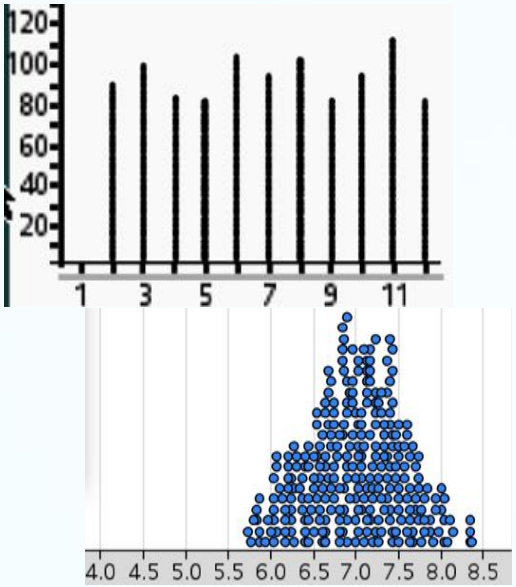
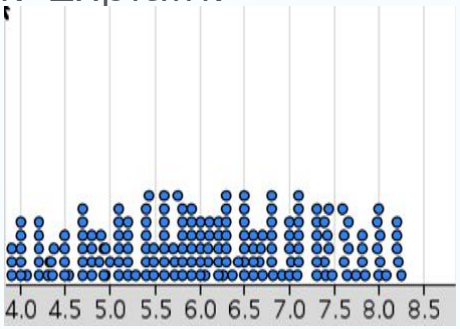


Figure 2

b) Which of the distributions below could be a simulated distribution of sample means for a sample of size 200 randomly selected from the original population? Explain.

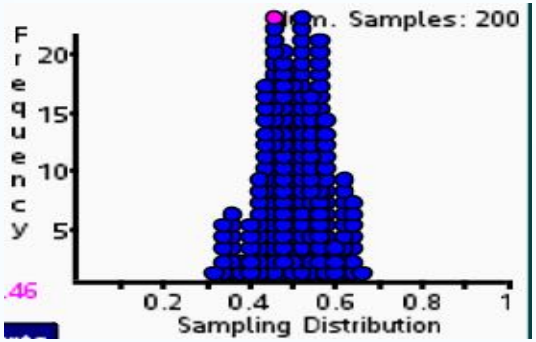
Figure 3



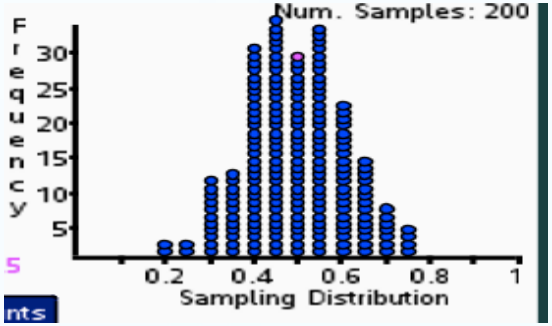
SOLO taxonomy level	Concept images for relating sampling distributions to populations and samples
Prestructural (P)	38% were still thinking about the distribution of the sample: "Figure 3 because again all of the number are being selected equally it appears and because for this scenario every number 2 to 12 has an equal chance of being selected." or "I think it could be figure 3 because the x-axis is labeled the same and there would have to be a lot of number taken form [sic] around 6-8 for it to be the figure 2."
Multistructural (M)	54% recognized that sampling distributions of sample means will be a normal distribution typically using the central limit theorem or similar language for their justification.

Which simulated distribution of sample proportions came from a sample of size 30 and which one from a sample of size 50?

a)



b)



Level of SOLO taxonomy	**Concept images for relating sampling distributions to populations and samples**
Prestructural (P)	29% associated variability with frequency and not sample size
Unistructural (U)	10% (recognized the variability was smaller but argued from number of samples
Multistructural (M)	
Relational (R)	52% linked sample size to variability, visually compared distribution of sample proportions with respect to sample size

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Features associated with a concept image related to random samples

SOLO taxonomy level	Concept images for sampling distributions: Randomness
Prestructural (P)	Represents random samples as those that are "accidental" or outcomes that happen without input from an observer
Unistructural (U)	Identifies sampling strategies that will produce a random sample; uses definition of random sample in specifying samples
Multistructural (M)	Connects randomness to bias and equal opportunity for selection; distinguishes between relative frequency and frequency distribution; interprets variability in random samples from the same population
Relational (R)	Connects randomness to short term unpredictability and long run predictability; relates to sampling distributions of a random variable

Choosing a random sample

- All students correctly identified three strategies that would produce random samples with acceptable reasoning related either to bias or describing why everyone would not have an equal chance to be selected (M).
- The fourth strategy: “To represent the population in a room, everyone chooses a number and those with even numbers are the sample.”
- 50%+ said yes, a random sample because everyone has “a 50% chance of choosing an even number” (P), three qualified their yes by saying that those in the room would have to draw a number from a bag or use another random device to assign the numbers.

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